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58. Proposed by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University, Miss.

An endless uniform chain is hung over two small smooth pegs in the same horizontal line. Show that, when it is in a position of equilibrium, the ratio of the distance between the vertices of the two catenaries to half the length of the chain is the tangent of half the angle of inclination of the portions near the pegs. [From *Routh's Analytical Statics. Mathematicl Tripos, 1855.*]

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Let h_1, h_2 be the vertical distances of the vertices of the two catenaries below the line $(=2a)$ joining the pegs ; then using the usual notation of the theory of the catenary, the origin being the middle of $2a$,

$$h_1+c_1-y=\tfrac{1}{2}c_1(e^{x/c_1}+e^{-(x/c_1)})\ldots\ldots(1),$$

$$s_1=\tfrac{1}{2}c_1(e^{x/c_1}-e^{-(x/c_1)})\ldots\ldots(2),$$

with like equations for the second curve.

From (1) and (2), α_1 being the inclination of the curve at the peg,

$$(dy/dx)=\tfrac{1}{2}(e^{x/c_1}-e^{-(x/c_1)})=\tan\alpha_1\ldots\ldots(3).$$

$$(ds_1/dx)=\tfrac{1}{2}(e^{x/c_1}+e^{-(x/c_1)})=\sec\alpha_1\ldots\ldots(4).$$

$$\therefore\ \sec\alpha_1+\tan\alpha_1=e^{a/c_1}\ldots\ldots(5).$$

$y=0$ in (1) gives $h_1+c_1=\frac{1}{2}c_1(e^{a/c_1}+e^{-(a/c_1)})=s_1/\sin\alpha_1\ldots\ldots(6)$,

by aid of (5), and by the usual theory, $c_1=s_1\cot\alpha_1\ldots\ldots(7)$, and then from (6) and (7),

$h_1=s_1\tan\frac{1}{2}\alpha_1$..(8). Similarly, $h_2=s_2\tan\frac{1}{2}\alpha_2$..(9), and $h_2+c_2=s_2/\sin\alpha_2$..(10).

For equilibrium, the right members of (6) and (10) are equal, since they measure the tensions in the two parts of the chain at either peg.

$$\therefore\ \frac{s_1}{s_2}=\frac{\sin\alpha_1}{\sin\alpha_2}=\frac{\cot\frac{1}{2}\alpha_2+\tan\frac{1}{2}\alpha_2}{\cot\frac{1}{2}\alpha_1+\tan\frac{1}{2}\alpha_1}\ldots\ldots(11).$$

Then finally,

$$\frac{h_1-h_2}{s_1+s_1}=\frac{(s_1/s_2)\tan\frac{1}{2}\alpha_1-\tan\frac{1}{2}\alpha_2}{(s_1/s_2+1}\ldots\ldots(12).$$

(11) in (12) gives

$$\frac{h_1-h_2}{s_1+s_1}=\tan\tfrac{1}{2}(\alpha_1-\alpha_2)\ldots\ldots(13).$$

II. Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, P. O. University, Miss., and HENRY HEATON, M. Sc., Atlantic, Iowa.

Let $2l$ be the length of the lower catenary, c its parameter, ψ the angle which its tangent at the right-hand peg makes with its directrix ; and let the same letters, accented, refer, similarly, to the upper catenary.

Then $\tan\psi=l/c$ and $\tan\psi'=l'/c'$, from which

$$\tan(\psi-\psi')=\frac{c'l-cl'}{cc'+ll'};\ \text{and}\ \tan\tfrac{1}{2}(\psi-\psi')=\frac{\sqrt{(c^2+l^2)(c'^2+l'^2)}-cc'-ll'}{c'l-cl'}.$$

The tension in the chain at any point being equal to the weight of as much of it as would reach from that point to the directrix, and the resultant tensions in the two catenaries at the pegs being the same, it follows that the two catenaries have a common directrix. Also, the square of the ordinate of any point equals the sum of the squares of the parameter and the length of the arc between the vertex and that point. From these facts, $l^2+c^2=l'^2+c'^2$. Substituting,

$$\tan\tfrac{1}{2}(\psi-\psi')=\frac{c^2+l^2-cc'-ll'}{c'l-cl'}=\frac{(c-c')c+(l-l')l}{c'l-cl'}.$$

Remembering that $(c-c')(c+c')=(l'-l)(l+l')$, this may be reduced to $(c'-c)/(l+l')$.

But $c'-c$ is the distance between the vertices of the two catenaries, $l+l'$ is half the length of the chain, and $\psi-\psi'$ is the inclination of the portions near the peg. Hence the proposition is established.

Also solved by *G. B. M. ZERR*.

DIOPHANTINE ANALYSIS.

57. **Proposed by JOSIAH H. DRUMMOND, LL. D., Counselor at Law, Portland, Maine.**

Each of *five* of the digits may be the terminal figure of a perfect integral square. Each of *eighteen* combinations of two digits may be the *two* terminal figures of an integral square. Each of *one hundred and nineteen* combinations of three digits may be the *three* terminal figures of an integral square. *Under these conditions*, what is the greatest number of arrangements of the nine digits, all taken together, whose three terminal figures shall be those of a square number?

Solution by the PROPOSER.

The first two conditions are embraced in the third and must be disregarded: the terminal figure of the last three must be one of five digits; and the two terminal figures must be one of the eighteen combinations of two digits. There are 119 combinations of three digits that may be terminal figures: with each one of these combinations, within the terms of the question, the other six digits may be used as many times as there are combinations in the six taken together, that is $2\times3\times4\times5\times6=720$. Hence $119\times720=85{,}680$, the number required.

59. **Proposed by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.**

Find the sum of the mth power of all the numbers less than P and prime to it, and then by substitution find the sum when $m=1, 2, 3, 4, 5$.

Solution by the PROPOSER.

The sum of the mth powers of all numbers less than P and not prime to it is given by the sum of